Minitest 4 - MTH 1410

Dr. Adam Graham-Squire, Fall 2017

Name: Key

9:41 ⇒ 35 min.

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 5 questions of the test, however you should still show all of your work. No calculators are allowed on the last question of the test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge above.
7. Number of questions = 5. Total Points = 30.

1. (5 points) Calculate the definite integral. Show your work and give an exact answer (No decimal approximation, though if you want to use your calculator to confirm that your answer is correct, a decimal approximation may be useful).

$$\int_1^4 \left(\frac{e^x}{3} - \frac{7}{x^3}\right) dx$$

$$\int_1^4 \left(\frac{1}{3}(e^x) - 7x^{-3}\right) dx \quad \checkmark$$

$$= \frac{1}{3} e^x \Big|_1^4 - 7\left(\frac{x^{-2}}{-2}\right)\Big|_1^4 \quad \checkmark \checkmark$$

$$= \frac{1}{3}e^4 - \frac{1}{3}e^1 - \left(\frac{7}{-2}(4^{-2}) - \frac{7}{-2}(1)^{-2}\right) \quad \checkmark \checkmark$$

$$= \boxed{\frac{e^4}{3} - \frac{e}{3} + \frac{7}{32} - \frac{7}{2}}$$

$$\begin{array}{r} 16 \\ 7 \\ \hline 112 \end{array}$$

$$= \boxed{\frac{e^4 - e}{3} - \frac{105}{32}}$$

2. (5 points) Let $g(x) = \int_3^x t^2(t-4)(t+2)dt$, where g is defined for all values of x. On what interval(s) is $g(x)$ increasing? You should not need a calculator to solve this problem, but you can use one if you think it will help you.

$g'(x) = \frac{d}{dx}\int_3^x t^2(t-4)(t+2)\,dt$

$g'(x) = x^2(x-4)(x+2)$ by F.T.C.

$0 = x^2(x-4)(x+2)$ 0.5

$\Rightarrow x = 0, 4, -2$ 0.5

+	−	−	+

$-2 \quad 0 \quad 4$

$g'(-3) = + \cdot - \cdot -$

$g'(-1) = + \cdot - \cdot +$

$g'(1) = + \cdot - \cdot +$

$g'(5) = +$

$\Rightarrow$ increasing on $(-\infty, -2)$ and $(4, \infty)$

No Calculator Name: Key

3. (5 points) Calculate the indefinite integral:

$$\int \left(\frac{x^2}{x^3} + \frac{3}{1+x^2} - (\csc x)(\cot x) \right) dx$$

0.5

$$= \int \left(\frac{1}{x} + 3\left(\frac{1}{1+x^2}\right) - (\csc x)(\cot x) \right) dx$$

$$= \boxed{\ln|x| + 3\tan^{-1}x + \csc x + C} \quad 0.5$$

4. (6 points) Calculate the indefinite integral:

$$\int x^3(3-2x^4)^3\,dx$$

$u = 3 - 2x^4$

$\frac{du}{dx} = -8x^3$

$\frac{du}{-8x^3} = dx$

$$= \int \cancel{x^3}\,(u)^3\left(\frac{du}{-8\cancel{x^3}}\right)$$

$$= \frac{-1}{8}\int u^3\,du \quad 0.5$$

$$= \frac{-1}{8}\cdot\frac{u^4}{4} + C \quad 0.5 \quad 0.5$$

$$= \boxed{\frac{-1}{32}(3-2x^4)^4 + C} \quad 0.5$$

or $\int x^3(9-12x^4+4x^8)(3-2x^4)\,dx$

$$= \int x^3(27 - 36x^4 + 12x^8 - 18x^4 + 24x^8 - 8x^{12})\,dx$$

$$= \int (27x^3 - 54x^7 + 36x^{11} - 8x^{15})\,dx$$

$$= \frac{27}{4}x^4 - \frac{54}{8}x^8 + \frac{36}{12}x^{12} - \frac{8}{16}x^{16} + C \quad 0.5 \quad 0.5$$

$$= \boxed{6.75x^4 - 6.75x^8 + 3x^{12} - \frac{x^{16}}{2} + C}$$

5. (9 points) Below, the first few steps are done for using the limit definition to calculate a definite integral. Answer the questions below (a brief answer is sufficient). You can use the blank next page if you need more room.

$$\begin{aligned}
\int_0^3 x^2\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x && (1)\\
&= \lim_{n\to\infty} \frac{3}{n}\left[f\left(\frac{3}{n}\right) + f\left(\frac{6}{n}\right) + f\left(\frac{9}{n}\right) + \cdots + f\left(\frac{3n}{n}\right)\right] && (2)\\
&= \lim_{n\to\infty} \frac{3}{n}\left[\left(\frac{3}{n}\right)^2 + \left(\frac{6}{n}\right)^2 + \left(\frac{9}{n}\right)^2 + \cdots + \left(\frac{3n}{n}\right)^2\right] && (3)\\
&= \lim_{n\to\infty} \frac{3}{n}\left(\frac{3}{n}\right)^2\left[1^2 + 2^2 + 3^2 + \cdots + n^2\right] && (4)
\end{aligned}$$

1.5 (a) Explain where the $\frac{3}{n}$ comes from (between lines 1 and 2).

1.5 (b) Explain why all of the terms are squared in line 3.

✓ (c) Explain what is happening between lines 3 and 4.

3 (d) Use the magic formula $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ to help you finish off calculating the limit.

2 (e) Use the Fundamental Theorem of Calculus (the evaluation theorem) to calculate $\int_0^3 x^2\,dx$ to double-check your answer from (d).

(a) $\frac{3}{n}$ is the width of each rectangle. 3 is the interval length and n is the # of rectangle,

(b) You are substituting $\frac{3}{n}$, $\frac{6}{n}$, etc into the function $f(x) = x^2$, so get $\left(\frac{3}{n}\right)^2$, etc.

(c) The $\left(\frac{3}{n}\right)^2$ is being factored out of the ~~[illegible]~~ brack $\left[\left(\frac{3}{n}\right)^2 + \left(\frac{6}{n}\right)^2 + \cdots\right]$, leaving you with $\left[1^2 + 2^2 + \cdots\right]$

$$(d) = \lim_{n\to\infty} \frac{\cancel{27}^{\,9}}{n^{\cancel{3}\,2}} \left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,2} \right)$$

$$= \lim_{n\to\infty} \frac{9(2n^2+3n+1)}{2n^2}$$

$$= \frac{\cancel{18n^2}^{\,9}}{\cancel{2n^2}} + \frac{27\cancel{n}}{2n^{\cancel{2}}} + \frac{9}{2n^2}$$

$$= \lim_{n\to\infty} 9 + \underbrace{\frac{27}{2n}}_{\to 0} + \underbrace{\frac{9}{2n^2}}_{\to 0}$$

$$= \boxed{9}$$

$$(e)\ \int_0^3 x^2\,dx = \frac{1}{3}x^3\Big|_0^3 = \frac{1}{3}(3^3) - \frac{1}{3}(0^3)$$

$$= \frac{27}{3} = \boxed{9}$$

Extra Credit(1 point) If $\int_0^{10} f(x)\,dx = 42$, $\int_0^3 f(x)\,dx = -7$, $\int_7^3 f(x)\,dx = -12$, and $\int_9^{10} 3f(x)\,dx = 60$, what is $\int_7^9 f(x)\,dx$?

$\hookrightarrow \int_3^7 f(x)\,dx = 12$

$\hookrightarrow \Rightarrow \int_9^{10} f(x)dx = 20$

$$\int_0^{10} f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^7 f(x)\,dx + \int_7^9 f(x)\,dx + \int_9^{10} f(x)\,dx$$

$$42 = -7 + 12 + \int_7^9 f(x)\,dx + 20$$

$$49 - 32 = \boxed{\int_7^9 f(x)\,dx = 17}$$